A General Formulation of Under-Actuated Manipulator Systems

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Abstract

This paper discusses a general formulation both in kinematics and dynamics for under-actuated manipulator systems, represented by a free-joint manipulator, a free-floating manipulator, a flexible-base manipulator and a flexible-arm manipulator. We highlight the fact that a concept, originally developed for a particular system, is applicable to the others as well because the dynamic equations of all four systems show common characteristics. Examples of such concepts are the generalized Jacobian matrix and the generalized inertia tensor which were originally developed for a free-floating manipulator, and also the reaction null-space concept, which was introduced for a free-floating and a flexible-base manipulator. We show also that the differences of the four systems are essentially due to the integrability conditions.

1 Introduction

The interest toward complex robot systems is expanding for new application areas. A class of such robot systems are under-actuated systems, characterized by the number of control actuators being less than the number of degree of freedom. Typical examples are serial manipulators with passive joints, free-floating space manipulators, flexible-arm manipulators and others. Jain and Rodriguez [1] have shown that the dynamics of various types of under-actuated systems can be formulated within a common framework. Currently, intensive research in the field is going on, using geometrical mechanics and differential geometry as the theoretical base [2][3].

Another example of an under-actuated system is a dextrous manipulator arm mounted on a passive flexible base (**Figure 2**). In literature, such a system is known under the name *long-reach manipulator* [4][5], or *flexible structure mounted manipulator system (FSMS)* [6]. A major issue concerning long-reach manipulators is dynamic interaction between the motion of the dextrous manipulator and the vibrational motion of the flex-

Figure 1 An example of Free-Floating Space Manipulator

Figure 2 An example of Flexible-Base Manipulator ible base due to induced reaction. One can find that the problem here is similar to the problem in free-floating manipulators in the sense that the dynamic reaction of the manipulator arm induces the interactive motion in the supporting base. The difference between these two systems is that the manipulator base of a free-floating manipulator is a floating inertia, but that of a long-reach manipulator is an inertia-spring-damper system.

One more example of an under-actuated system is a flexible-arm manipulator, the system DOF of which is a sum of the DOF of controllable joints and the DOF of elastic displacements. The joint motion and elastic displacements are dynamically interacting in some fashion similar to other under-actuated systems. Compared with long-reach manipulators which have a concentrated compliance at the base, flexible-arm manipulators have distributed compliance along the kinematic chain.

Figure 3 shows a family of under-actuated manipulator systems. The top two represent systems with *free* joints, while the bottom two show manipulators with *compliant* or visco-elastic joints. Since a flexible-link manipulator can be modeled as a serial kinematic chain composed of active and elastic joints alternately, we regard a flexible-*arm* manipulator and a flexible-*joint* manipulator as systems of the same type. In the figure, the left column is a group of *separable* systems, which means that free or compliant (passive) joints are separately located from active joints. The right column is a group of *distributed* systems, where passive joints are distributed among other active joints.

The above systems have been studied independently although, they eventually show a lot of similarity in the dynamic formulation. For example, a free-floating manipulator is a system composed by a free base (which can be modeled as a 6 DOF free joint in general) and active manipulator(s). But once the free joints are distributed, the system can be regarded as a serial manipulator with free joints, and both systems basically have common dynamics. A long-reach manipulator is a system composed by a compliant base (which can be modeled as a 6 DOF visco-elastic beam in general) and active manipulator(s). But when the compliant joints are distributed, the system can be regarded as a manipulator with compliant joints or a flexible-arm manipulator.

If we look at differences among these four systems, non-holonomy plays a significant role. A free-floating manipulator is known to be generally a first-order non-holonomic system [7]. On the other hand, a free-joint manipulator can be holonomic or non-holonomic depending on its physical construction. For example, it is known for a 2R horizontal manipulator that the one with the 1st joint free and the 2nd active is holonomic, but the other with the 1st joint active and the 2nd free is non-holonomic [8]. As far as flexible-base and flexible-arm manipulators are concerned, we consider them non-holonomic in general, but here we will discuss some special conditions for their integrability.

The main purpose of this paper is to compare the various types of under-actuated systems, highlighting the fact that a concept originally developed for a particular system is applicable to the others as well. The paper is organized in three parts: In the first part, we focus on a general formulation for under-actuated manipulator systems, paying attention to the fact that all the systems in this class have common characteristics in the dynamic formulation. We use a matrix-based approach and show that the Generalized Jacobian Matrix [9][10] and the Generalized Inertia Tensor [11]–[12], which were originally developed for a free-floating space manipulator, are applicable to other systems as well. In the second part, we examine the characteristics of constraint dynamics for the above four types of under-actuated systems, case by case. Here, the differences of the dynamic character are discussed from the point of view of holonomy or integrability. In the last part, we focus on some further common concepts, such as the measure of dynamic coupling, the kinematic compensability, and the reaction null-space.

2 General Formulation

Let us consider a system whose motion is described by n degrees of freedom of the generalized coordinate $\boldsymbol{q} \in R^n$ for *active* joints and m degrees of freedom of the generalized coordinate $\boldsymbol{p} \in R^m$ for *passive* joints. Define $\mathcal{F}_q$ as active generalized force generated on coordinate $\boldsymbol{q}$, and $\mathcal{F}_p$ as a passive generalized force on coordinate $\boldsymbol{p}$. Also, define $\boldsymbol{x}$ as a coordinate of a point of interest (the operational space coordinate), and let an external wrench $\mathcal{F}_{ex}$ be applied on $\boldsymbol{x}$. This wrench i mapped as $\boldsymbol{J}_q^T \mathcal{F}_{ex}$ and $\boldsymbol{J}_p^T \mathcal{F}_{ex}$ onto each generalized coordinate respectively, using corresponding Jacobian matrices. The equation of motion of

Figure 3 A family of under-actuated manipulator systems

such system is expressed as:

$$\begin{bmatrix} \boldsymbol{H}_p & \boldsymbol{H}_{pq} \\ \boldsymbol{H}_{pq}^T & \boldsymbol{H}_q \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{p}} \\ \ddot{\boldsymbol{q}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{c}_p \\ \boldsymbol{c}_q \end{bmatrix} + \begin{bmatrix} \boldsymbol{g}_p \\ \boldsymbol{g}_q \end{bmatrix} = \begin{bmatrix} \mathcal{F}_p \\ \mathcal{F}_q \end{bmatrix} + \begin{bmatrix} \boldsymbol{J}_p^T \\ \boldsymbol{J}_q^T \end{bmatrix} \mathcal{F}_{ex} \quad (1)$$

where $\boldsymbol{H}_p \in R^{m\times m}$, $\boldsymbol{H}_q \in R^{n\times n}$, $\boldsymbol{H}_{pq} \in R^{m\times n}$ are inertia matrices, $\boldsymbol{c}_p$, $\boldsymbol{c}_q$ non-linear Coriolis and centrifugal forces, and $\boldsymbol{g}_p$, $\boldsymbol{g}_q$ gravity terms.

The kinematic relationship among $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{x}$ is expressed as:

$$\dot{\boldsymbol{x}} = \boldsymbol{J}_p\dot{\boldsymbol{p}} + \boldsymbol{J}_q\dot{\boldsymbol{q}}, \quad (2)$$

$$\ddot{\boldsymbol{x}} = \boldsymbol{J}_p\ddot{\boldsymbol{p}} + \dot{\boldsymbol{J}}_p\dot{\boldsymbol{p}} + \boldsymbol{J}_q\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}_q\dot{\boldsymbol{q}}. \quad (3)$$

The upper and lower sets of Equation (1) are written as:

$$\boldsymbol{H}_p\ddot{\boldsymbol{p}} + \boldsymbol{H}_{pq}\ddot{\boldsymbol{q}} + \boldsymbol{c}_p + \boldsymbol{g}_p = \mathcal{F}_p + \boldsymbol{J}_p^T\mathcal{F}_{ex}, \quad (4)$$

$$\boldsymbol{H}_q\ddot{\boldsymbol{q}} + \boldsymbol{H}_{pq}^T\ddot{\boldsymbol{p}} + \boldsymbol{c}_q + \boldsymbol{g}_q = \mathcal{F}_q + \boldsymbol{J}_q^T\mathcal{F}_{ex}. \quad (5)$$

Since $\mathcal{F}_q$ is an actively controlled force, while $\mathcal{F}_p$ is a resultant "passive" one, Equation (4) works as a dynamic constraint to the system. If $\boldsymbol{H}_p$ is non-singular, the acceleration $\ddot{\boldsymbol{p}}$ of the passive coordinate is:

$$\ddot{\boldsymbol{p}} = -\boldsymbol{H}_p^{-1}(\boldsymbol{H}_{pq}\ddot{\boldsymbol{q}} + \boldsymbol{c}_p + \boldsymbol{g}_p - \mathcal{F}_p - \boldsymbol{J}_p^T\mathcal{F}_{ex}) \quad (6)$$

By substituting it into Equation (5), we obtain the equation of motion involving the acceleration $\ddot{\boldsymbol{q}}$ of the active joints:

$$\widehat{\boldsymbol{H}}\ddot{\boldsymbol{q}} + \widehat{\boldsymbol{c}} + \widehat{\boldsymbol{g}} = \widehat{\mathcal{F}} + \widehat{\boldsymbol{J}}^T\mathcal{F}_{ex} \quad (7)$$

where

$$\widehat{\boldsymbol{H}} = \boldsymbol{H}_q - \boldsymbol{H}_{pq}^T\boldsymbol{H}_p^{-1}\boldsymbol{H}_{pq}, \quad (8)$$

$$\widehat{\boldsymbol{J}} = \boldsymbol{J}_q - \boldsymbol{J}_p\boldsymbol{H}_p^{-1}\boldsymbol{H}_{pq}, \quad (9)$$

$$\widehat{\boldsymbol{c}} = \boldsymbol{c}_q - \boldsymbol{H}_{pq}^T\boldsymbol{H}_p^{-1}\boldsymbol{c}_p, \quad (10)$$

$$\widehat{\boldsymbol{g}} = \boldsymbol{g}_q - \boldsymbol{H}_{pq}^T\boldsymbol{H}_p^{-1}\boldsymbol{g}_p, \quad (11)$$

$$\widehat{\mathcal{F}} = \mathcal{F}_q - \boldsymbol{H}_{pq}^T\boldsymbol{H}_p^{-1}\mathcal{F}_p. \quad (12)$$

The matrix $\widehat{\boldsymbol{H}}$ is known as the *generalized inertia tensor* [11].

The matrix $\widehat{\boldsymbol{J}}$ is known as the *generalized Jacobian matrix* [9][10]. It was originally defined for a free-floating manipulator under the condition of momentum conservation. But here, we see that the generalized Jacobian exists regardless of this condition. Hence, the generalized Jacobian can be defined for any under-actuated system.

Equations (10)(11)(12) have the same structure: the forces $\boldsymbol{c}_p$, $\boldsymbol{g}_p$ and $\mathcal{F}_p$ related to the passive variables are mapped onto the active variable space through the dynamic coupling expression $\boldsymbol{H}_{pq}^T\boldsymbol{H}_p^{-1}$.

On the other hand, if the passive acceleration (6) is substituted into the kinematic equation (3), the following is obtained:

$$\begin{aligned} \ddot{\boldsymbol{x}} &= (\boldsymbol{J}_q - \boldsymbol{J}_p\boldsymbol{H}_p^{-1}\boldsymbol{H}_{pq})\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}_p\dot{\boldsymbol{p}} + \dot{\boldsymbol{J}}_q\dot{\boldsymbol{q}} \\ &\quad -\boldsymbol{J}_p\boldsymbol{H}_p^{-1}(\boldsymbol{c}_p + \boldsymbol{g}_p - \mathcal{F}_p - \boldsymbol{J}_p^T\mathcal{F}_{ex}) \\ &\equiv \widehat{\boldsymbol{J}}\ddot{\boldsymbol{q}} + \ddot{\boldsymbol{\zeta}}. \end{aligned} \quad (13)$$

The non-linear acceleration $\ddot{\boldsymbol{\zeta}}$ can be computed from a model or measurement. Then the inverse kinematic solution is given as:

$$\ddot{\boldsymbol{q}} = \widehat{\boldsymbol{J}}^{-1}(\ddot{\boldsymbol{x}} - \ddot{\boldsymbol{\zeta}}). \quad (14)$$

The effectiveness of the generalized Jacobian matrix in the inverse kinematic problems has been discussed only for a free-floating robot so far, but the above equation suggests the importance of the generalized Jacobian for other classes of under-actuated systems as well.

3 Case Formulation

Based on the general formulation, we derive the equation of motion for the following under-actuated systems: free-joint manipulators, free-floating manipulators, flexible-base manipulators and flexible-arm manipulators.

3.1 Free-Joint Manipulator

A free-joint manipulator is a serial link manipulator composed of active and passive joints. For this system replace the subscript notation in Equation (1) as follows:

p	$\rightarrow$	ϕ_p	:passive joint angle
q	$\rightarrow$	ϕ_a	:active joint angle
x	$\rightarrow$	x_h	:position/orientation of the hand
$\mathcal{F}_p$	$\rightarrow$	O	:passive joint torque
$\mathcal{F}_q$	$\rightarrow$	τ	:active joint torque
$\mathcal{F}_{ex}$	$\rightarrow$	O	:external wrench on the hand

The dynamic equation becomes:

$$\begin{bmatrix} \boldsymbol{H}_p & \boldsymbol{H}_{pa} \\ \boldsymbol{H}_{pa}^T & \boldsymbol{H}_a \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{\phi}}_p \\ \ddot{\boldsymbol{\phi}}_a \end{bmatrix} + \begin{bmatrix} \boldsymbol{c}_p \\ \boldsymbol{c}_a \end{bmatrix} + \begin{bmatrix} \boldsymbol{g}_p \\ \boldsymbol{g}_a \end{bmatrix} = \begin{bmatrix} \boldsymbol{O} \\ \boldsymbol{\tau} \end{bmatrix} \tag{15}$$

The characteristics of the dynamic constraint provided by the upper set of (15) has been recently studied for integrability. According to [8], the condition of existence of a first integral is

A-1. the gravity force $\boldsymbol{g}_p$ is constant,

A-2. the passive joint angles $\boldsymbol{\phi}_p$ do not appear in the manipulator inertia matrix.

The condition of existence of a second integral is

B-1. a first integral exists,

B-2. the distribution defined by the null-space of matrix $\begin{bmatrix} \boldsymbol{H}_p & \boldsymbol{H}_{pa} \end{bmatrix}$ is involutive.

For example, a 2R horizontal manipulator with the 1st joint free and the 2nd active is holonomic, but one with the 1st joint active and the 2nd free is non-holonomic.

3.2 Free-Floating Manipulator

A free-floating manipulator is composed of a floating base and a serial articulated arm. Assume no gravity and zero external forces. Then replace the subscript notation in Equation (1) as follows:

p	$\rightarrow$	x_b	:position/orientation of the base
q	$\rightarrow$	ϕ	:joint angle of the arm
x	$\rightarrow$	x_h	:position/orientation of the hand
$\mathcal{F}_p$	$\rightarrow$	O	:generalized force on the base
$\mathcal{F}_q$	$\rightarrow$	τ	:joint torque of the arm
$\mathcal{F}_{ex}$	$\rightarrow$	O	:external wrench on the hand

We obtain the following equations:

$$\begin{bmatrix} \boldsymbol{H}_b & \boldsymbol{H}_{bm} \\ \boldsymbol{H}_{bm}^T & \boldsymbol{H}_m \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{x}}_b \\ \ddot{\boldsymbol{\phi}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{c}_b \\ \boldsymbol{c}_m \end{bmatrix} = \begin{bmatrix} \boldsymbol{O} \\ \boldsymbol{\tau} \end{bmatrix} \tag{16}$$

$$\dot{\boldsymbol{x}}_h = \boldsymbol{J}_m \dot{\boldsymbol{\phi}} + \boldsymbol{J}_b \dot{\boldsymbol{x}}_b \tag{17}$$

$$\ddot{\boldsymbol{x}}_h = \boldsymbol{J}_m \ddot{\boldsymbol{\phi}} + \dot{\boldsymbol{J}}_m \dot{\boldsymbol{\phi}} + \boldsymbol{J}_b \ddot{\boldsymbol{x}}_b + \dot{\boldsymbol{J}}_b \dot{\boldsymbol{x}}_b \tag{18}$$

In the equation of motion, the gravity is zero and the inertia matrices $\boldsymbol{H}_b \in R^{6\times 6}$, $\boldsymbol{H}_m \in R^{n\times n}$, $\boldsymbol{H}_{bm} \in R^{6\times n}$ are functions of $\boldsymbol{\phi}$ only. Thus, the conditions A-1 and A-2 are satisfied. Hence, the constraint dynamic equation

$$\boldsymbol{H}_b \ddot{\boldsymbol{x}}_b + \boldsymbol{H}_{bm} \ddot{\boldsymbol{\phi}} + \dot{\boldsymbol{H}}_b \dot{\boldsymbol{x}}_b + \dot{\boldsymbol{H}}_{bm} \dot{\boldsymbol{\phi}} = \boldsymbol{O} \tag{19}$$

is integrated to yield the system momentum:

$$\mathcal{L} = \boldsymbol{H}_b \dot{\boldsymbol{x}}_b + \boldsymbol{H}_{bm} \dot{\boldsymbol{\phi}} = const. \tag{20}$$

$\mathcal{L}$ involves linear and angular momentum: linear momentum is once more integrable to yield the position of the system centroid, while angular momentum is not anymore integrable because the condition B-2 is not satisfied. Therefore, a free-floating manipulator is a first-order non-holonomic system [7].

Solving (20) for $\dot{\boldsymbol{x}}_b$ and substituting into (17), we obtain a useful kinematic equation with the generalized Jacobian matrix.

$$\dot{\boldsymbol{x}}_h = \widehat{\boldsymbol{J}} \dot{\boldsymbol{\phi}} + \dot{\boldsymbol{x}}_{h0} \tag{21}$$

where

$$\widehat{\boldsymbol{J}} = \boldsymbol{J}_m - \boldsymbol{J}_b \boldsymbol{H}_b^{-1} \boldsymbol{H}_{bm} \tag{22}$$

and

$$\dot{\boldsymbol{x}}_{h0} = \boldsymbol{J}_b \boldsymbol{H}_b^{-1} \mathcal{L} \tag{23}$$

Since $\boldsymbol{H}_b$ is the inertia tensor of a single rigid body (the manipulator base), it is always positive definite and its inverse exists. Note that Equation (22) is the original definition of the generalized Jacobian matrix. For a first-order non-holonomic system, the generalized Jacobian can be derived from the velocity equations.

In the presence of an external wrench $\mathcal{F}_h$ at the manipulator hand, we get the following dynamic equation, by analogy to Equation (7):

$$\widehat{\boldsymbol{H}} \ddot{\boldsymbol{\phi}} + \widehat{\boldsymbol{c}} = \boldsymbol{\tau} + \widehat{\boldsymbol{J}}^T \mathcal{F}_h \tag{24}$$

where

$$\widehat{\boldsymbol{H}} = \boldsymbol{H}_m - \boldsymbol{H}_{bm}^T \boldsymbol{H}_b^{-1} \boldsymbol{H}_{bm}. \tag{25}$$

An external wrench at the hand occurs when a free-floating manipulator collides or catches an object, and at this moment, the system momentum changes. If the definition of $\widehat{\boldsymbol{H}}$ and $\widehat{\boldsymbol{J}}$ relies on the momentum conservation, we cannot use these matrices for the impact modeling. But as discussed in the previous section, Equation (24) is generally valid regardless of the momentum conservation. This fact justifies the modeling and analysis of impact dynamics [12][13] based on Equation (24).

3.3 Flexible-Base Manipulator

Next, consider a flexible-base manipulator composed by a single manipulator base which is constrained by a flexible-beam or a spring and damper (visco-elastic) system, and a serial manipulator arm at whose endpoint an external wrench may apply. For such a flexible-base manipulator, replace the subscript notation in Equation (1) as follows:

p	$\rightarrow$	x_b	:position/orientation of the base
q	$\rightarrow$	ϕ	:joint angle of the arm
x	$\rightarrow$	x_h	:position/orientation of the endpoint
$\mathcal{F}_p$	$\rightarrow$	$\mathcal{F}_b$	:forces to deflect the flexible base
$\mathcal{F}_q$	$\rightarrow$	τ	:joint torque of the arm
$\mathcal{F}_{ex}$	$\rightarrow$	$\mathcal{F}_h$	:external wrench on the endpoint

Then we obtain the following equations:

$$\begin{bmatrix} \boldsymbol{H}_b & \boldsymbol{H}_{bm} \\ \boldsymbol{H}_{bm}^T & \boldsymbol{H}_m \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{x}}_b \\ \ddot{\boldsymbol{\phi}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{c}_b \\ \boldsymbol{c}_m \end{bmatrix} + \begin{bmatrix} \boldsymbol{g}_b \\ \boldsymbol{g}_m \end{bmatrix} = \begin{bmatrix} \mathcal{F}_b \\ \boldsymbol{\tau} \end{bmatrix} + \begin{bmatrix} \boldsymbol{J}_b^T \\ \boldsymbol{J}_m^T \end{bmatrix} \mathcal{F}_h \quad (26)$$

$$\dot{\boldsymbol{x}}_h = \boldsymbol{J}_m \dot{\boldsymbol{\phi}} + \boldsymbol{J}_b \dot{\boldsymbol{x}}_b \quad (27)$$

$$\ddot{\boldsymbol{x}}_h = \boldsymbol{J}_m \ddot{\boldsymbol{\phi}} + \dot{\boldsymbol{J}}_m \dot{\boldsymbol{\phi}} + \boldsymbol{J}_b \ddot{\boldsymbol{x}}_b + \dot{\boldsymbol{J}}_b \dot{\boldsymbol{x}}_b \quad (28)$$

The difference from the previous two classes of under-actuated systems is the existence of the base constraint force $\mathcal{F}_b$. Let $\boldsymbol{D}_b, \boldsymbol{K}_b$ be damping and spring factors of the flexible-base. Then, the constraint force $\mathcal{F}_b$ is expressed as:

$$\mathcal{F}_b = -\boldsymbol{D}_b \dot{\boldsymbol{x}}_b - \boldsymbol{K}_b \Delta \boldsymbol{x}_b \quad (29)$$

where $\Delta \boldsymbol{x}_b$ denotes elastic base displacement from the equilibrium position.

The upper set of Equation (26) is rearranged as:

$$\boldsymbol{H}_b \ddot{\boldsymbol{x}}_b + \boldsymbol{D}_b \dot{\boldsymbol{x}}_b + \boldsymbol{K}_b \Delta \boldsymbol{x}_b = -\boldsymbol{g}_b - \mathcal{F}_m + \boldsymbol{J}_b^T \mathcal{F}_h \quad (30)$$

where

$$\mathcal{F}_m = \boldsymbol{H}_{bm} \ddot{\boldsymbol{\phi}} + \dot{\boldsymbol{H}}_{bm} \dot{\boldsymbol{\phi}} \quad (31)$$

is the dynamic reaction force induced by the manipulator. Equations (26) and (30) are familiar expressions for flexible-base manipulators [14] [15].

The constraint dynamic equation (30) is not integrable, i.e. it imposes a non-holonomic constraint, generally. But let us consider some special cases when we can find an integral.

Suppose that $\mathcal{F}_h = \boldsymbol{O}$ and the gravity $\boldsymbol{g}_b = \boldsymbol{k}_1$ is constant. Then Equation (30) is expressed as:

$$\mathcal{F}_b + \mathcal{F}_m + \boldsymbol{k}_1 = \boldsymbol{O}. \quad (32)$$

Here we also assume that $\mathcal{F}_b$ does not contain $\boldsymbol{\phi}$ and has a constant value, and that $\mathcal{F}_m$ does not contain $\boldsymbol{x}_b$ and has a constant value. Under such condition, Equation (32) can be integrated to yield

$$\mathcal{V}(t) + \mathcal{L}_m + \boldsymbol{k}_1 t + \boldsymbol{k}_2 = \boldsymbol{O} \quad (33)$$

where $\mathcal{V}(t)$ is a function to express the vibration around a dynamic equilibrium given by Equation (32). Also,

$$\mathcal{L}_m = \boldsymbol{H}_{bm} \dot{\boldsymbol{\phi}} = \ell_1 t + \ell_2 \quad (34)$$

is the momentum of the manipulator part, which is termed the *coupling momentum* [16]. If there are enough degrees of freedom in the manipulator, i.e. not less than the number of passive coordinates, $n \geq 6$ for this case, we can find manipulator motions to satisfy the above integrable condition.

A reduced form of the dynamic equation is derived as:

$$\widehat{\boldsymbol{H}} \ddot{\boldsymbol{\phi}} + \widehat{\boldsymbol{c}} + \widehat{\boldsymbol{g}} = \boldsymbol{\tau} + \widehat{\boldsymbol{J}}^T \mathcal{F}_h + \boldsymbol{R} \mathcal{F}_b \quad (35)$$

where $\boldsymbol{R} = \boldsymbol{H}_{bm}^T \boldsymbol{H}_b^{-1}$. This will be useful for force control with explicit measurement of both $\mathcal{F}_h$ and $\mathcal{F}_b$ wrenches.

3.4 Flexible-Arm Manipulator

Finally, consider a manipulator system composed by flexible links. A flexible link has an infinite number of vibrational DOF (modes) actually, but it can be approximately modeled as a successive chain of a finite number of virtual elastic joints [17][18]. The formulation has exactly the same structure as other under-actuated systems. This is apparent by replacing the subscript notation in Equation (1) as follows:

p	$\rightarrow$	e	:elastic deflection of the flexible links
q	$\rightarrow$	ϕ	:active joint angle
x	$\rightarrow$	x_h	:position/orientation of the endpoint
$\mathcal{F}_p$	$\rightarrow$	$\mathcal{F}_e$	:forces to deflect the flexible links
$\mathcal{F}_q$	$\rightarrow$	τ	:joint torque
$\mathcal{F}_{ex}$	$\rightarrow$	$\mathcal{F}_h$	:external wrench on the endpoint

Then we obtain the following equations:

$$\begin{bmatrix} \boldsymbol{H}_e & \boldsymbol{H}_{em} \\ \boldsymbol{H}_{em}^T & \boldsymbol{H}_m \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{e}} \\ \ddot{\boldsymbol{\phi}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{c}_e \\ \boldsymbol{c}_m \end{bmatrix} + \begin{bmatrix} \boldsymbol{g}_e \\ \boldsymbol{g}_m \end{bmatrix} = \begin{bmatrix} \mathcal{F}_e \\ \boldsymbol{\tau} \end{bmatrix} + \begin{bmatrix} \boldsymbol{J}_e^T \\ \boldsymbol{J}_m^T \end{bmatrix} \mathcal{F}_h \quad (36)$$

$$\dot{\boldsymbol{x}}_h = \boldsymbol{J}_m \dot{\boldsymbol{\phi}} + \boldsymbol{J}_e \dot{\boldsymbol{e}} \quad (37)$$

$$\ddot{\boldsymbol{x}}_h = \boldsymbol{J}_m\ddot{\boldsymbol{\phi}} + \dot{\boldsymbol{J}}_m\dot{\boldsymbol{\phi}} + \boldsymbol{J}_e\ddot{\boldsymbol{e}} + \dot{\boldsymbol{J}}_e\dot{\boldsymbol{e}} \tag{38}$$

The force of elastic deflection is generally expressed with stiffness and damping matrices as:

$$\mathcal{F}_e = -\boldsymbol{D}_e\dot{\boldsymbol{e}} - \boldsymbol{K}_e\Delta\boldsymbol{e} \tag{39}$$

where $\Delta\boldsymbol{e}$ denotes the elastic displacement from the equilibrium position.

Here we can have the same discussion on integrability as for the flexible-base manipulator. But the difference is that the number of elastic coordinates is generally larger than the number of manipulator joints ($m > n$). Hence, for any given end-effector path, we cannot find a manipulator motion which would satisfy the integrability condition. This non-holonomic nature renders the inverse path tracking control problem difficult.

4 Common Concepts

In the above sections, we showed that the four types of under-actuated systems have a common structure in the equations of motion, and their differences are found in the integrability conditions. In this section, we focus on some concepts which were originally proposed and developed for one class of under-actuated systems, but are applicable to any other class as well. More specifically, we will discuss the measure of dynamic coupling, kinematic compensability, and the reaction null-space concept.

4.1 Measure of Dynamic Coupling

Dynamic coupling between the active coordinate and the passive coordinate is evaluated in some papers [19][20]. Torres [19] derived a simple expression for the displacement of the flexible beam induced by the manipulator reaction under a quasi-statically condition as:

$$\delta\boldsymbol{x}_b = -\boldsymbol{H}_b^{-1}\boldsymbol{H}_{bm}\delta\boldsymbol{\phi}, \tag{40}$$

which lead to the *Coupling Map* of flexible-base manipulators.

Following this expression, and using the general notation of Equation (1), we obtain:

$$\delta\boldsymbol{p} = -\boldsymbol{H}_p^{-1}\boldsymbol{H}_{pq}\delta\boldsymbol{q}. \tag{41}$$

The matrix product $\boldsymbol{H}_p^{-1}\boldsymbol{H}_{pq}$ appears in Equations (7)-(11) to represent the dynamic coupling property. For example, when the expression $\boldsymbol{H}_p^{-1}\boldsymbol{H}_{pq}$ appearing in (7) and (8) is a null matrix (e.g. due to specific inertia and mass properties), then the generalized Jacobian matrix and the generalized inertia tensor coincide with the Jacobian and inertia tensor of a "conventional" manipulator. As the eigenvalues of $\boldsymbol{H}_p^{-1}\boldsymbol{H}_{pq}$ get larger, the dynamic coupling effects become significant. It would be appropriate to have a measure of such coupling. We propose the following one:

$$I_d = \sqrt{\det\left(\boldsymbol{H}_p^{-1}\boldsymbol{H}_{pq}\boldsymbol{H}_{pq}^T\boldsymbol{H}_p^{-T}\right)} \tag{42}$$

4.2 Kinematic Compensability

The idea of compensability is originally proposed for a flexible-arm manipulator [21] and for a system composed by flexible-macro and rigid-micro manipulators [22]. Kinematic compensability can be defined via Equation (37), when the elastic deflections are slightly perturbed from their equilibrium position:

$$\delta\boldsymbol{x}_h = \boldsymbol{J}_m\delta\boldsymbol{\phi} + \boldsymbol{J}_e\delta\boldsymbol{e}. \tag{43}$$

When the elastic deflection $\delta\boldsymbol{e}$ is totally compensated by the joint action $\delta\boldsymbol{\phi}$, we can get $\delta\boldsymbol{x}_h = 0$. The amount of compensation is

$$\delta\boldsymbol{\phi} = -\boldsymbol{J}_m^+\boldsymbol{J}_e\delta\boldsymbol{e} \tag{44}$$

Such a compensation is possible, only when

$$\text{range}(\boldsymbol{J}_m) \supseteq \text{range}(\boldsymbol{J}_e). \tag{45}$$

The degree of compensability can be evaluated through the measure [18]:

$$I_k = \frac{1}{\sqrt{\det\left(\boldsymbol{J}_m^+\boldsymbol{J}_e\boldsymbol{J}_e^T\boldsymbol{J}_m^{+T}\right)}}. \tag{46}$$

The above equations are applicable to any type of under-actuated system, by replacing the subscript notation as:

$$\boldsymbol{e} \to \boldsymbol{p}, \quad \boldsymbol{\phi} \to \boldsymbol{q}, \quad \boldsymbol{x}_h \to \boldsymbol{x}$$

$$\boldsymbol{J}_e \to \boldsymbol{J}_p, \quad \boldsymbol{J}_m \to \boldsymbol{J}_q$$

For example, the application to a flexible-base manipulator is straightforward, considering the situation that the deformation of the flexible-beam is compensated by the joint displacement.

For manipulators with free-joints or a floating-base, an interesting interpretation is obtained in terms of the so-called *manipulator inversion* [23]. The manipulator can be operated to change the angles of the free-joints (or the base), keeping the endpoint displacement $\delta\boldsymbol{x} = 0$.

Note that the fixed-attitude-restricted dexterity proposed in [23] and expressed in our general notation as:

$$I_{FAR} = \sqrt{\det\left[\boldsymbol{J}_q(\boldsymbol{E} - \boldsymbol{H}_{pq}^{+}\boldsymbol{H}_{pq})\boldsymbol{J}_q^T\right]} \qquad (47)$$

is a closely related concept. $\boldsymbol{E} \in R^{n\times n}$ is the identity matrix. This index is derived through a kinematic relationship excluding the passive coordinates:

$$\dot{\boldsymbol{x}} = \boldsymbol{J}_q(\boldsymbol{E} - \boldsymbol{H}_{pq}^{+}\boldsymbol{H}_{pq})\dot{\boldsymbol{q}}. \qquad (48)$$

4.3 Reaction Null-Space

The reaction null-space is a useful idea to discuss the coupling and decoupling of dynamic interaction between the active coordinates and the passive coordinates. This concept has its roots in the earlier work on free-floating space manipulator by Nenchev et al, where the Fixed-Attitude-Restricted (FAR) Jacobian has been proposed as means to plan [23] and control [24] manipulator motion that does not disturb the attitude of the free-floating base. Application of the reaction null-space with relation to impact dynamics can be found in [13]. In a recent study [6] the authors emphasized the fact that the reaction null-space concept is general, and can be applied to a broad class of moving base manipulators.

Now let us recall the definition of the reaction null-space in terms of our general notation. In the system described by Equations (1), or (4) and (5), let us consider the case when the external wrench at endpoint $\mathcal{F}_{ex} = \boldsymbol{O}$, say free-space motion of the manipulator. without any contact with the environment. If the constraint equation (4) is integrable and the coupling momentum $\mathcal{L}_m$ is defined with a constant value, then the manipulator reaction $\mathcal{F}_m = \boldsymbol{O}$ and no reaction force or torque is induced on the passive coordinates. In case the number of DOF of the active coordinates is larger than that of the passive ones $n > m$, the solution for the manipulator operation to satisfy $\bar{\mathcal{L}}_m = const$ is given by:

$$\dot{\boldsymbol{q}} = \boldsymbol{H}_{pq}^{+}\bar{\mathcal{L}}_m + (\boldsymbol{E} - \boldsymbol{H}_{pq}^{+}\boldsymbol{H}_{pq})\dot{\boldsymbol{\xi}} \qquad (49)$$

where $\dot{\boldsymbol{\xi}} \in R^n$ is an arbitrary vector.

The projector $(\boldsymbol{E} - \boldsymbol{H}_{pq}^{+}\boldsymbol{H}_{pq})$ maps vectors onto the null space of the inertia matrix $\boldsymbol{H}_{pq}$. This is an inertial null space, which we termed *reaction null-space* .

In the special case of zero initial condition $(\bar{\mathcal{L}}_m = \boldsymbol{O})$, we obtain:

$$\dot{\boldsymbol{q}}_{ns} = (\boldsymbol{E} - \boldsymbol{H}_{pq}^{+}\boldsymbol{H}_{pq})\dot{\boldsymbol{\xi}}. \qquad (50)$$

As long as we operate the manipulator joints using the joint velocities given by (50), no reaction force or torque is generated on the base, therefore no reactive motion or vibration is observed in the passive coordinates. The integration of (50) yields *reactionless paths* which are joint space trajectories not exciting the motion of the passive coordinates.

On the other hand, the first term in Equation (49) suggests maximum interaction with the base, which is due to the properties of the pseudoinverse. This maximum interaction characteristics can be used, for example, to effectively damp out the motion of the passive coordinates. Using the measurement of the passive displacement $\Delta\boldsymbol{p}$ as a feedback signal and G as a gain matrix, we have a simple, but effective damping control law:

$$\dot{\boldsymbol{q}}_v = -G\boldsymbol{H}_{pq}^{+}\Delta\boldsymbol{p} \qquad (51)$$

The above control space is perpendicular to the reaction null-space. Therefore these two operations (50) and (51) can be easily superimposed without interfering each other, just by simple addition.

$$\dot{\boldsymbol{q}}_c = \dot{\boldsymbol{q}}_v + \dot{\boldsymbol{q}}_{ns}$$
$$\dot{\boldsymbol{q}}_c = -G\boldsymbol{H}_{pq}^{+}\Delta\boldsymbol{p} + (\boldsymbol{E} - \boldsymbol{H}_{pq}^{+}\boldsymbol{H}_{pq})\dot{\boldsymbol{\xi}} \qquad (52)$$

The availability and effectiveness of the reaction null-space has been successfully demonstrated by simulations and experiments, as shown in the following section. For a planar 2R flexible-base manipulator, $n = 2$ and $m = 1$ then it is relatively easy to obtain the reaction null-space solutions, yielding the reactionless paths. On the other hand, a spatial flexible-base or floating-base manipulator is characterized with $m = 6$ in general. But the number of passive coordinates can be artificially restricted, for example, considering base position only, or base attitude only (i.e. $m = 3$), then it would be possible to make use of the reaction null-space concept and the reactionless paths. Such a possibility exists also without the artificial restriction, but with a kinematically redundant manipulator (i.e. $n > 6$). On the other hand, for flexible-arm manipulators we have $m > n$ in general and the reaction null space does not exist. Only when the number of elastic coordinates (or vibration modes) is limited and less than the number of active joints, then it would be possible to use the reaction null space.

5 Experiments and Simulations of Reactionless Operation

One of the remarkable outcomes from the discussion in this paper is that there exists a unique operation to yield zero reaction to the passive coordinates in the under-actuated manipulator systems. In this section, we shall highlight such an operation, called *reactionless operation*, evidenced by experiments and numerical simulations.

5.1 Planar 2R Experiments

A planar experimental setup of a flexible-base manipulator, called TREP, is developed in Tohoku University [25]. The setup consists of a small 2R rigid link manipulator attached to the free end of a flexible double beam representing an elastic base (see **Figure 4**). The manipulator is driven by DC servomotors with velocity command input.

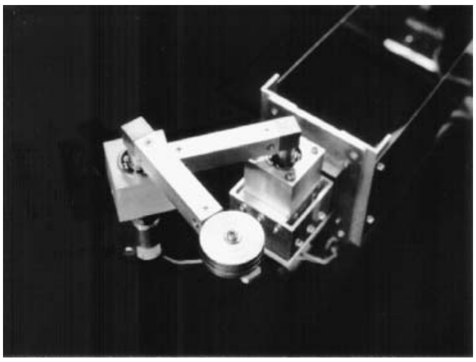

Figure 4 Photo of the experimental setup TREP

The TREP setup is modeled according to **Figure 5**. Since the elastic base has been designed as a double beam, the torsion of the beam can be neglected as a disturbance. This is also the case with the reaction force component along the longitudinal axis of the base. Thus, we shall consider just the reaction force along the so-called low stiffness direction, which coincides with the x axis of the elastic-base coordinate frame. This means that $m = 1$. Since the manipulator has two motors ($n = 2$), the reaction null-space is one-dimensional, meaning that there is one nonzero vector in the reaction null space. Recalling the

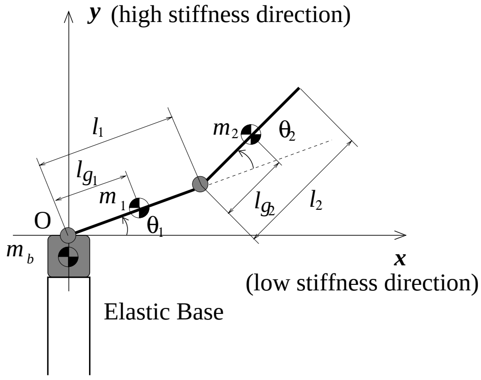

Figure 5 The model of TREP

definition of the coupling momentum (34), we obtain as equation for our experimental setup.

$$\mathcal{L}_m = \boldsymbol{h}_{bm} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \qquad (53)$$

where $\boldsymbol{h}_{bm} = \begin{bmatrix} h_{bm1} & h_{bm2} \end{bmatrix}$ denotes the coupling matrix, with

$$h_{bm1} = -(m_1 l_{g1} + m_2 l_1) sin(\theta_1) - m_2 l_{g2} \sin(\theta_1 + \theta_2),$$

$$h_{bm2} = -m_2 l_{g2} \sin(\theta_1 + \theta_2).$$

The reaction null space vector becomes then

$$\boldsymbol{n} = \begin{bmatrix} h_{bm2} & -h_{bm1} \end{bmatrix}^T. \qquad (54)$$

A zero initial coupling momentum will be conserved with any joint velocity parallel to the reaction null space vector. This vector induces a one-dimensional distribution in joint space, which, with well-conditioned coupling, is integrable. Consequently, the set of reactionless paths of the system can be obtained. This set is displayed in **Figure 6**.

We have conducted experiments of reactionless operation by tracking a reactionless path. **Figure 7** shows the experimental results, obtained with the initial configuration $\theta_1 = \theta_2 = 0$ where the arm was extended and aligned with the flexible base. The trace of the end-point is depicted in the upper part of the figure. This operation is generated by Equation (50) with the corresponding coupling inertia shown above. The (scalar) velocity on the path was determined from the variable $\dot{\xi}$, which was designed as a fifth-order spline function of time. In order to verify the possibility for an arbitrary choice of $\dot{\xi}$, we performed the motion on the same path twice, with different velocities

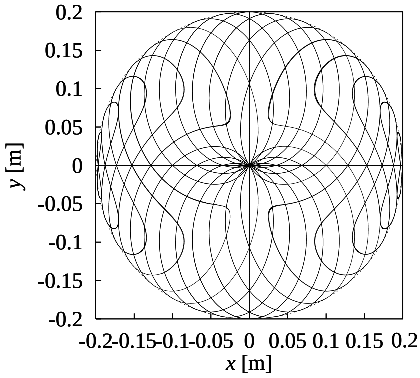

Figure 6 Reactionless paths of TREP

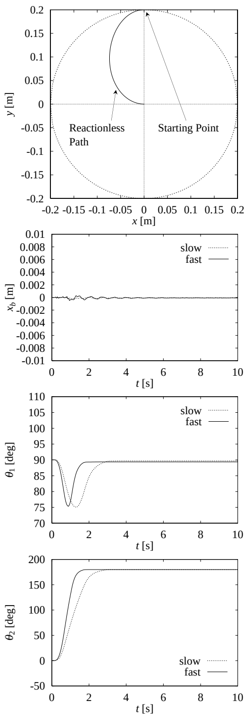

Figure 7 Reactionless operation

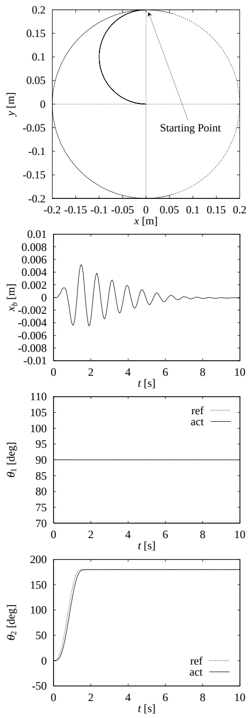

Figure 8 Point-to-point operation

(called fast and slow). The lower three graphs in the same figure show these two results. It is seen that almost no base vibration is excited in both cases, and in spite of the significant difference in the joint velocity.

For comparison, **Figure 8** shows a point-to-point motion path (in fact, just joint two rotation) with the same boundary conditions as in the previous motion. The base vibrates significantly.

5.2 Simulation of a 7DOF Flexible-Base Manipulator

We shall expand the application target to realistic 3 dimensional systems. One of the examples is a flexible-base manipulator, composed by a 7DOF rigid manipulator and a flexible structure in arbitrary shape. Such an example is found in manipulators used on a space station, e.g. the Space Station Remote Manipulator System (SSRMS) topped with Special Purpose Dextrous Manipulator (SPDM). The SSRMS, a macro arm, is a long-reach manipulator used for the relocation of the SPDM, a micro arm. The SSRMS plays the role of a supporting base for the micro arm when performing fine manipulation tasks. Note, however, that a space long-reach arm is usually designed as a light-weight arm. Thus, it comprises inherent flexibility which implies that vibrations will be excited due to the reaction from the micro arm. These vibrations are significant and degrade the operational accuracy and efficiency.

In order to avoid the excitation of vibrations, the end-point of the macro arm, i.e. the base of the micro arm should yield zero reactions in all 6DOF directions (3DOF translation and 3DOF orientation.) Such an operation will be possible if the arm has more than 6DOF and the reaction null-space control is applied.

We developed a dynamic simulation model of a 7DOF manipulator arm mounted on an L-shaped flexible structure (see **Figure 2**.) In the simulation, we use joint torque control (not velocity control) to realize the reactionless operation. The control torque is computed by substituting Equation (50) into the lower set of Equation (26), to obtain:

$$\boldsymbol{\tau} = \tilde{\boldsymbol{G}}_f \dot{\boldsymbol{x}}_b + \tilde{\boldsymbol{c}} + \boldsymbol{H}_m \boldsymbol{n} \tag{55}$$

where

$$\tilde{\boldsymbol{G}}_f = -\boldsymbol{H}_m \boldsymbol{H}_{bm}^+ \boldsymbol{G}_f,$$

$$\tilde{\boldsymbol{c}} = \boldsymbol{c}_m - \boldsymbol{H}_m \boldsymbol{H}_{bm}^+ \boldsymbol{c}_b,$$

$$\boldsymbol{n} = (\boldsymbol{E} - \boldsymbol{H}_{bm}^+ \boldsymbol{H}_{bm})\boldsymbol{\xi}.$$

$\boldsymbol{G}_f$ denotes a constant feedback gain matrix and $\boldsymbol{n} \in R^n$ is an arbitrary vector from the reaction null space. This control law guarantees that system damping is achieved, resulting in zero vibrations or vibration suppression if any [16]. Here we assume zero gravity and no external forces at the manipulator end-point.

Figure 9 depicts one of the reactionless operations obtained by Equation (55). The upper graph shows joint operation of the arm. The lower shows resulting vibrations in the base structure, in this case no vibrations at all. During the operation, joint 7 makes several turns because it has relatively small moment of inertia. For comparison, **Figure 10** depicts a PD feedback operation to yield the same initial and final joint angles as **Figure 9,** except for joint 7. Since this operation doesn't take care of the reaction, the base vibrates significantly.

5.3 Simulation of a 6DOF Floating-Base Manipulator

A typical example of a practical floating-base manipulator is the Experimental Test Satellite-VII (ETS-VII), developed and successfully launched by the National Space Development Agency of Japan, NASDA. The satellite carries a 2 meter-long 6DOF manipulator arm on a 2,500 kilogram satellite (see **Figure 1**.) 6DOF is not enough to have zero reaction in all directions at the base. However, for a satellite flying in orbit, translational deviation is less significant than attitude deviation. For the attitude deviation, if the satellite turns by 0.5 degrees, the communication link for manipulator operation will be disabled. We therefore pay attention to 3DOF attitude reaction of the base only, thus obtain a 3DOF reaction null-space with a conventional 6DOF manipulator.

We applied a control law similar to that in Equation (55), and obtained reactionless operation. **Figure 11** depicts the results. The upper graph shows joint operation of the arm. The lower shows the resulting base motion, in this case, almost zero attitude deviation while the translation is non-zero. For comparison, **Figure 12** depicts a conventional operation to yield the same initial and final joint angles as in **Figure 11.** In this case, the attitude deviation of the base becomes too significant to maintain the communication link.

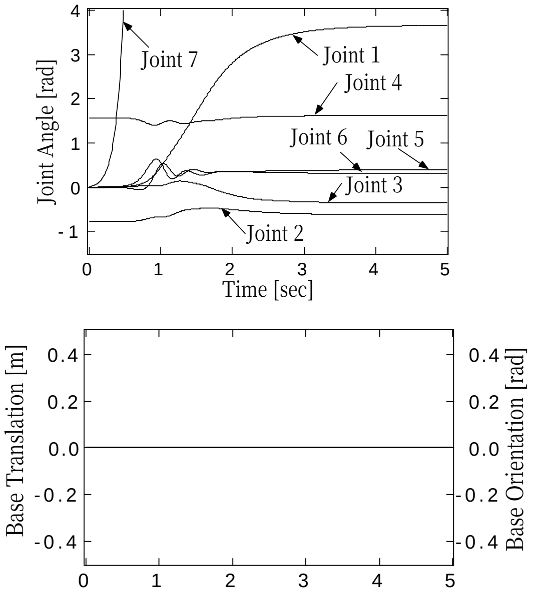

Figure 9 Reactionless operation for 7DOF flexible-base manipulator

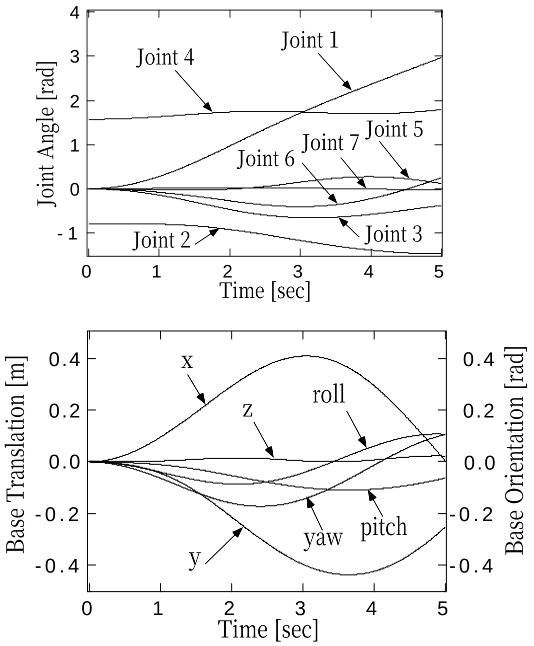

Figure 10 Conventional operation in 7DOF flexible-base manipulator

6 Conclusion

This paper discussed a general formulation both in kinematics and dynamics for under-actuated manipulator systems, represented by a free-joint manipulator, a free-floating manipulator, a flexible-base manipulator and a flexible-arm manipulator. We highlighted the fact that a concept originally developed for a particular system is applicable to the other systems as well, because the dynamic equations show common characteristics. Examples of such concepts are the generalized Jacobian matrix and the generalized inertia tensor which were originally developed for a free-floating manipulator. Another example is the reaction null-space developed for free-floating and flexible-base manipulators. We also emphasized

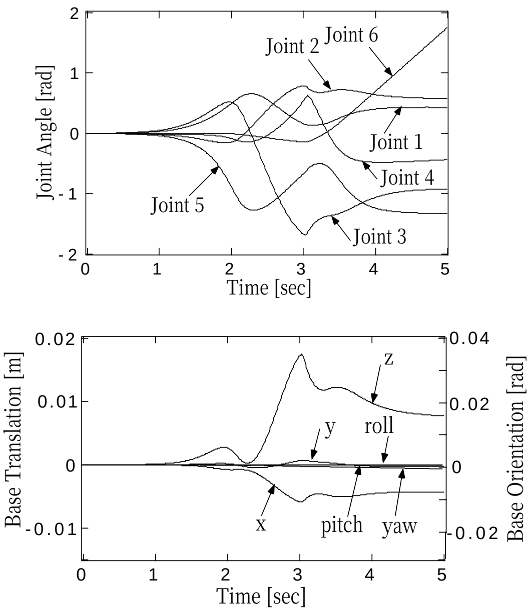

Figure 11 Reactionless operation for 6DOF floating-base manipulator

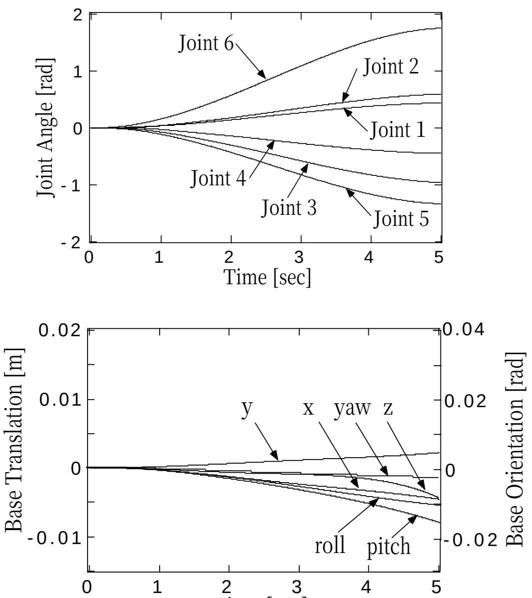

Figure 12 Conventional operation in 6DOF floating-base manipulator

the fact that differences among the four systems come from integrability conditions.

As one of the remarkable outcomes of this paper, the reactionless operation is emphasized by experiments and numerical simulations for flexible-base and floating-base manipulators. Such operations might be useful in space robotic missions, where the structural vibrations and the base attitude disturbance due to manipulator reaction will significantly degrade the quality and efficiency of the missions.

From the point of view of common dynamic characteristics, this paper suggested some new applications, including (a) force controls using Equation (7) for every manipulator, (b) inverse kinematic operations based on the generalized Jacobian (14) for free-joint, flexible-base and flexible-arm manipulators, and (c) tasks using the reaction null-space for free-joint and flexible-arm manipulators in case of $n > m$; these three problems are left for future study.

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